of the fluid layer boundary. If the fluid flow in the shell is not stratified, then as we know, the system loses stability for significantly higher stream velocities ($V_* > V_{kn}$, flutter).

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REFLECTION AND TRANSMISSION OF SOUND WAVES THROUGH THE INTERFACIAL BOUNDARY OF TWO JOINED ELASTIC HALF-STRIPS*

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A numerical solution of the problem of the incidence of plane harmonic waves on the interfacial boundary of two joined half-strips with different elastic properties is presented. A detailed analysis is given of the reflection and transmission of the incident wave energy through the interfacial boundary, and the nature of the state of stress and strain is investigated in its neighbourhood. The wave fields in longitudinally inhomogeneous media were studied earlier in /1-3/ etc.

1. We examine an infinite strip of thickness $2h$. We connect it to an x, z Cartesian system of coordinates such that the z axis is orthogonal to the strip boundaries while the x axis coincides with its middle line. Let the plane boundary $x = 0$ be the line separating the properties of the material, and let λ_k, μ_k, ρ_k be the elastic moduli and the density of the material to the left of the interfacial boundary $(x < 0, k = 1)$ and to the right of it $(x > 0, k = 2)$. We shall assume the boundaries of the strip $z = \pm h$ to be stress-free.

We introduce the four-dimensional vector $\mathbf{W} = (u, w, \sigma, \tau)^T$ characterizing the wave field in the strip into the consideration. Here $u = u_x$, $w = u_z$ are the displacement vector components and $\sigma = \sigma_{xx}$, $\tau = \tau_{xz}$ are the corresponding stress tensor components.

Let a plane normal Lamb wave of unit amplitude $\mathbf{W}_j^{(1)}(z, \gamma_j^{(1)}) \exp[i(\gamma_j^{(1)}x - \Omega t)]$ be incident from $x = -\infty$ onto the interfacial boundary, where $\Omega = \omega h/c$ is the dimensionless frequency ($c = \max\{\sqrt{\mu_1/\rho_1}, \sqrt{\mu_2/\rho_2}\}$), $\gamma_j^{(1)}$ is the j-th wave number related to Ω by the Rayleigh-Lamb dispersion equation /4/, and $\mathbf{W}_j^{(1)}$ is a four-dimensional vector whose components are determined for compression-tension waves by the relationships

$$u_j = i\gamma_j(\operatorname{ch}\alpha_1 z - S_j\alpha_2^{-1}\operatorname{ch}\alpha_2 z)$$
$$w_j = \gamma_j^2\alpha_1^{-1}\operatorname{sh}\alpha_1 z - S_j\operatorname{sh}\alpha_2 z$$
$$\tau_j = \mu\left(\frac{\partial u_j}{\partial z} + i\gamma_j w_j\right), \quad \sigma_j = i\gamma_j(\lambda + 2\mu)u_j + \lambda\frac{\partial w_j}{\partial z}$$
$$\alpha_1^2 = \gamma_j^2 - \frac{\rho\Omega^2}{\mu}, \quad \alpha_2^2 = \gamma_j^2 - \frac{\rho\Omega^2}{\lambda + 2\mu}$$
$$S_j = \frac{\alpha_1^2 + \gamma_j^2}{2\alpha_1}\frac{\operatorname{sh}\alpha_1 h}{\operatorname{sh}\alpha_2 h}$$

The superscript $k = 1, 2$ in parentheses indicates that the quantities belong to the medium located, respectively, to the left or the right of the interfacial boundary of the material

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properties.

In the usual notion the wave field in the first and second half-strips, taking the radiation conditions into account, can be represented in the form

$$\mathbf{W}^{(1)} = \mathbf{W}_j^{(1)}(z, \gamma_j^{(1)}) \exp[i(\gamma_j^{(1)}x - \Omega t)] + \Sigma A_{1n} \mathbf{W}_n^{(1)}(z, -\gamma_j^{(1)}) \exp[i(-\gamma_n^{(1)}x - \Omega t)] \tag{1.1}$$

$$\mathbf{W}^{(2)} = \Sigma A_{2n} \mathbf{W}_n^{(2)}(z, \gamma_n^{(2)}) \exp[i(\gamma_n^{(2)}x - \Omega t)]$$

$$\operatorname{Im} \gamma_l^{(k)} \geqslant 0; \quad d\Omega/d\gamma_l^{(k)} > 0 \ (\operatorname{Im} \gamma_l^{(k)} = 0)$$

Here and henceforth the summation is from $n = 1$ to $n = \infty$.

Remark. For definite values of Ω when $d\Omega/d\gamma = 0$ the Rayleigh-Lamb equation has multiple eigenvalues. In this case additional components corresponding to associated eigenfunctions occur in the relationships (1.1). See /5/ for more details.

A_{1n} and A_{2n} in (1.1) are the amplitudes of the reflected and transmitted waves. The waves which propagate correspond to real $\gamma_l^{(k)}$ and inhomogeneous waves localized near the $x = 0$ boundary and decaying with distance from it correspond to complex $\gamma_l^{(k)}$.

Taking account of the relationships (1.1), the continuity condition for the displacements and the stress vector on the interfacial boundary

$$\mathbf{W}^{(1)} = \mathbf{W}^{(2)}, \quad x = 0 \tag{1.2}$$

can be written in the form (the factor $\exp(-i\Omega t)$ is omitted)

$$\mathbf{W}_j^{(1)}(\gamma_j^{(1)}) + \Sigma A_{1n} \mathbf{W}_n^{(1)}(-\gamma_n^{(1)}) = \Sigma A_{2n} \mathbf{W}_n^{(2)}(\gamma_n^{(2)}) \tag{1.3}$$

We use the generalized orthogonality condition /6, 7/ to reduce the functional equations (1.3) to algebraic equations

$$(\mathbf{W}_l^{(k)}, U\mathbf{W}_m^{(k)}) \equiv \int_{-h}^{h} [\sigma_l^{(k)} \bar{u}_m^{(k)} + \tau_l^{(k)} \bar{w}_m^{(k)} - \bar{\sigma}_m^{(k)} u_l^{(k)} - \bar{\tau}_m^{(k)} w_l^{(k)}]\, dz = 4i\Omega^{-1}\delta_{lm} P_l^{(k)} \tag{1.4}$$

$$P_l^{(k)} = 0, \quad \operatorname{Im}\gamma_l^{(k)} \neq 0; \quad P_l^{(k)} \neq 0, \quad \operatorname{Im}\gamma_l^{(k)} = 0; \quad U = \begin{Vmatrix} 0 & -E \\ E & 0 \end{Vmatrix}$$

(O is the zeroth and E the unit 2x2 matrix).

The quantity $P_l^{(k)}$ in (1.4) determines the mean energy flux per period transported through the transverse section $x = \text{const}$ by a wave with the wave number $\gamma_l^{(k)}$.

Multiplying relationship (1.3) scalarly successively by $U\mathbf{W}_m^{(1)}(-\gamma_m^{(1)})$, $U\mathbf{W}_m^{(2)}(\gamma_m^{(2)})$ and using condition (1.4), we arrive at an infinite pairwise system of algebraic equations

$$4i\Omega^{-1}P_m^{(1)} A_{1m} = \Sigma b_{nm} A_{2n}, \quad 4i\Omega^{-1}P_m^{(2)} A_{2m} = -\Sigma \bar{b}_{mn} A_{1n} + (\mathbf{W}_j^{(1)}(\gamma_j^{(1)}), U\mathbf{W}_m^{(2)}(\gamma_m^{(2)})) \tag{1.5}$$

$$(b_{nm} = (\mathbf{W}_n^{(2)}(\gamma_n^{(2)}), U\mathbf{W}_m^{(1)}(-\gamma_m^{(1)})), \quad m = 1, 2, \ldots)$$

The method of reduction is used to solve system (1.5), here the sufficiency criterion for the reduction was the degree of compliance with the matching conditions (1.2).

Besides all travelling waves, up to four pairs of inhomogeneous waves in each of the half-strips were taken into account in the formation of the systems in practical computations. The error in complying with the conjugate conditions in the displacement did not exceed 1% for the maximum quantity $u_x(z)$ in the whole frequency band considered. The accuracy in satisfying the matching conditions in the stresses for a given number of inhomogeneous waves taken into account in the interval $|z| < h(1 - 2\alpha)$ (α is the singularity index in the stresses at angular points that occur for definite unions of the media making contact /8/) did not exceed 5-6% of the maximum value of $\sigma_{xx}(z)$ in the incident wave.

When there are singularities the series in the stresses in the segment $h(1 - 2\alpha) \leqslant z \leqslant h$ diverge for $x = 0$ and generalized methods of summation and regularization /9, 10/ must be drawn upon to satisfy the matching conditions in the whole $|z| \leqslant h$ interval.

After the values of the amplitudes have been found from the infinite system, the equality of the incident wave energy to the sum of the reflected and transmitted wave energies is verified, which was an additional criterion for the reliability of the results obtained.

2. The values of the amplitudes A_{1n}, A_{2n} found from the reduced system permit complete determination of the wave fields in both half-strips and execution of an energy analysis of the process of wave reflection and transmission through the boundary $x = 0$.

Dispersion curves of each of the half-strips and the frequency dependence of the reflection coefficient

$$k_r = \frac{1}{P_1^{(1)}} \sum_{m=1}^{N} |A_{1m}|^2 P_m^{(1)}$$

are represented in Fig.1 (the ratio between the reflected and incident wave energies N is the number of waves propagating at this frequency) in the case of symmetric waveguide oscillations during incidence of the first normal compression-tension wave on the interfacial boundary. The dispersion curves for the left half-strip are displayed by solid lines for $\lambda_1 = 206$ GN/m^2, $\mu_1 = 153$ GN/m^2, $\rho_1 = 18.7 \times 10^3$ kg/m^3 and by dashes for the right half-strip for $\lambda_2 = 59.2$ GN/m^2, $\mu_2 = 32.6$ GN/m^2, $\rho_2 = 2.7 \times 10^3$ kg/m^3.

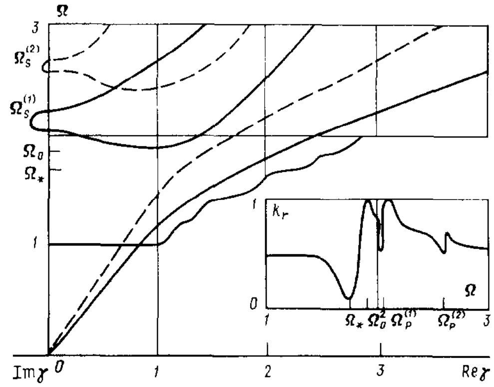

Fig.1

The Ω_*, $\Omega_0^{(k)}$, $\Omega_p^{(k)}$, $\Omega_s^{(k)}$ in Fig.1 are certain characteristic frequencies of the process of wave reflection through the boundary. The frequencies $\Omega_0^{(k)}$, $\Omega_p^{(k)}$, $\Omega_s^{(k)}$ are determined by the dispersion curves for the k-th strip. The group velocity vanishes at the frequency $\Omega_0^{(k)}$ while $\Omega_p^{(k)}$, $\Omega_s^{(k)}$ correspond to the thickness-stretch and the thickness-shear resonance frequencies and are determined from the known formulas /4/

$$\Omega_p^{(k)} = \frac{\pi(2m+1)}{2h} \sqrt{\frac{\lambda_k + 2\mu_k}{\rho_k}}, \quad \Omega_s^{(k)} = \frac{\pi m}{h} \sqrt{\frac{\mu_k}{\rho_k}},$$
$$m = 0, 1, 2, \ldots$$

The frequency Ω_* is determined numerically for each pair of materials and lies in the domain when just one travelling wave, comprising approximately 0.85-0.9 of the quantity $\Omega_0 = \min\{\Omega_0^{(1)}, \Omega_0^{(2)}\}$ propagates in the reflected and transmitted field.

Certain general regularities in the behaviour of k_r are clarified on the basis of a series of computations executed for different combinations of elastic properties of the materials making contact.

For $\Omega < \Omega_*$ the reflection coefficient is constant over the whole band in practice and is identical with the value obtained on the basis of computations by bar theory. An abrupt decrease in the coefficient k_r is observed upon approaching the frequency Ω_* which reaches its minimum value for $\Omega = \Omega_*$. The latter is related to the fact that a significant increase in the modulus of the amplitude of the first reflected non-propagating wave $|A_{12}|$ occurs at this frequency.

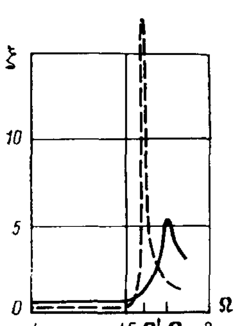

Fig.2

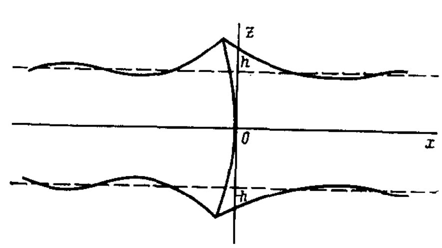

Fig.3

The ratio $\xi = |A_{12}/A_{11}|$ of the amplitude moduli of the reflected first inhomogeneous and propagating waves as a function of the frequency is represented in Fig.2 by a solid line for the mentioned pair of materials. The magnitude of the maximum of this ratio depends very much on the elastic properties of the materials making contact. Thus, if the material properties of the first half-strip remain unchanged, while softer material, say, with the moduli $\lambda_2 =$ 2.29 BN/m^2, $\mu_2 = 1.53$ GN/m^2, $\rho_2 = 93.5$ kg/m^3, is selected as material for the second half-strip, then the first non-propagating mode is excited much more significantly at the frequency Ω_*' (the dashed line in Fig.2). The frequency Ω_*' here is shifted somewhat relative to Ω^*, which is related to the change in the parameters, and is close to the edge resonance frequency for the first half-strip /4/.

The occurrence of intense oscillations localized in the neighbourhood of the half-strip interfacial boundary is characteristic for the behaviour of the solution at the frequency Ω_*. The computed mode of the oscillations in the neighbourhood of the half-strip interfacial boundary at the frequency Ω_*, which it is natural to call the boundary resonance frequency, is represented in Fig.3. The concept of boundary resonance can here be considered as a natural extension of the edge resonance concept to the case of two adjacent waveguides.

The magnitude of the minimum of the reflection coefficient at the frequency Ω_* obviously depends on the material parameters of the joined half-strips. For the first pair of materials considered above more than 96% of the incident wave energy is transported into the second medium for $\Omega = \Omega_*$ since this fraction comprises approximately 48% (Fig.1) for $\Omega < \Omega_*$. For the second pair of materials more than 60% of the energy is transported into the second medium at the frequency $\Omega = \Omega_*$ while practically all the energy (96%) is reflected from the interfacial boundary in the domain of the frequency $\Omega < \Omega_*$.

The next characteristic feature in the behaviour of the reflection coefficient is its increase on approaching the frequency Ω_0, starting with which three travelling waves appear in the reflected or transmitted field. The quantity k_r reaches its maximum value, approaching one, at the frequency Ω_0, which corresponds to practically total reflection of the incident wave energy.

As the frequency increases further, the behaviour of k_r is noticeably more complicated and depends substantially on the mutual arrangement of the dispersion curves, particularly on the mutual arrangement of the thickness-stretch $\Omega_p^{(k)}$ and thickness-shear $\Omega_s^{(k)}$ resonance frequencies $(k = 1, 2)$. Of the clarified regularities in the behaviour of the reflection coefficient at $\Omega > \Omega_0$ we note the presence of local minima of k_r in the neighbourhood of the low thickness-stretch resonances $\Omega_p^{(1)}$ and $\Omega_p^{(2)}$ (Fig.1).

The complexity of the behaviour of k_r in the frequency domain when several travelling waves start to propagate in the waveguide is manifest in the redistribution of the energy transported by each wave when the frequency changes. The percentage incident wave energy distribution between the reflected and transmitted travelling waves participating in the wave process (shown on parentheses) is presented in the table in different frequency bands for the pair of materials considered above.

It is seen from the table that the first travelling wave is the most energy filled in the frequency domain considered in the transmitted field while a mutual reflected wave energy redistribution is observed in the reflected field. Thus, the energy content of the first wave gradually drops with the appearance of three propagating reflected waves $\Omega_0^{(1)} < \Omega < \Omega_p^{(1)}$, $\Omega > \Omega_s^{(1)}$ while the third wave increases and it becomes most energy-filled in the reflected field. It should be noted here that in the frequency band in which the third travelling wave has opposite phase and group velocity signs, the energy redistribution depends substantially on Poisson's ratio ν_1 in the first medium. Thus if $\nu_1 > 1/3$ (in this case $\Omega_p^{(1)} > \Omega_s^{(1)}$) then the energy-content of the third travelling wave increases somewhat but does not exceed the energy content of the first.

The analysis performed for energy reflection and transmission through the interfacial boundary of half-strips over a broad frequency band enables us to conclude that for a definite pair of materials the greatest transmission of energy from one medium to another for it is observed at the boundary resonance frequency Ω_*. Practically total energy transmission through the boundary of a composite waveguide is observed here at this frequency for many combinations of materials in contact.

Frequency range	Ω	1	2	3
$\Omega < \Omega_*$	0.6	48.3(51.7)	—	—
Ω_*	1.76	3.9(96.1)	—	—
Ω_*, $\Omega_0^{(1)}$	1.84	40.6(59.4)	—	—
$\Omega_0^{(1)}$, $\Omega_p^{(1)}$	1.9	86.1(0.4)	6.2	7.3
	1.92	42.8(6.8)	14.6	35.8
	2.01	3.6(20.6)	15.4	60.4
$\Omega_p^{(1)}$, $\Omega_s^{(1)}$	2.04	26.5(47.2)	26.3	—
	2.2	88.9(6.6)	4.5	—
$\Omega_s^{(1)}$, $\Omega_p^{(2)}$	2.23	45.3(16.9)	5.9	31.9
	2.44	3.3(28.3)	2.7(1.1)	63.4(1.2)
$\Omega_p^{(2)}$, $\Omega_s^{(2)}$	2.67	0.1(34.9)	1.5(2.0)	61.5(—)
$\Omega > \Omega_s^{(2)}$	3.0	3.45(43.7)	1.45(1.7)	49.3(0.4)

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